Answer Sheet to the Written Exam

Corporate Finance and Incentives

December 2016

In order to achieve the maximal grade 12 for the course, the student must excel in all four problems.

The four problems jointly seek to test fulfillment of the course's learning outcomes: "After completing the course, the student should be able to:

Knowledge:

1. Understand, account for, define and identify the main methodologies, concepts and topics in Finance
2. Solve standard problems in Finance, partly using Excel
3. Criticize and discuss the main models in Finance, relating them to current issues in financial markets and corporate finance

Skills:

1. Manage the main topics and models in Finance
2. Organize material and analyze given problems, assessing standard models and results
3. Argue about financial topics, putting results into perspective, drawing on the relevant knowledge of the field

Competencies:

1. Bring into play the achieved knowledge and skills on new formal problems, and on given descriptions of situations in financial markets or corporations
2. Be prepared for more advanced models and topics in Finance."

Problems 1–3 are particularly focused on knowledge points 1 and 2, skills of type 1 and 2, competencies 1 and 2. Problem 4 emphasises knowledge points 1 and 3, skills 1 and 3, an competency 1.

Some numerical calculations may differ slightly depending on the software used for computation, so a little slack is allowed when grading the answers.

Problem 1 (CAPM 25%)

1) To solve $Az = \mathbf{1}$, note that $z = A^{-1}\mathbf{1}$ and use matrix inversion in Excel. The solution for z is $(3.99, 2.98, 1.32)^T$, normalized to the minimum-variance portfolio $x_m = (0.48, 0.36, 0.16)^T$.

2) To solve $Az = b - r_f\mathbf{1}$, note that $z = A^{-1}(b - r_f\mathbf{1})$ and use matrix inversion in Excel. The solution for z is $(0.12, 0.17, 0.11)^T$, normalized to the tangent portfolio $x_e = (0.30, 0.43, 0.28)^T$.

3) For the minimum-variance portfolio, the expected return is $x_m^T b = 0.058$ and the variance is $x_m^T A x_m = 0.121$. For the efficient portfolio, the expected return is $x_e^T b = 0.067$ and the variance is $x_e^T A x_e = 0.143$.

4) The desired expected return is 0.062. The simplest way to find the desired portfolio is to appeal to the Two Mutual Fund Theorem: The average portfolio $(x_m + x_e)/2$ is on the efficient frontier, and it has the desired expected return. This portfolio is $(0.39, 0.39, 0.22)^T$ and the variance on its return is 0.126.

Problem 2 (Debt and Equity 20%)

1) In the low state, the firm has only 80 million Kroner, but has promised 100 million Kroner to its creditors. They get all the 80 million Kroner. The value of the debt is then $D = (0.4(100) + 0.6(80))/1.03 = 85.4$ million Kroner. The value of equity is $.4(40)/1.03 = 15.5$ million Kroner. The firm's value is $D + E = 101.0$ million Kroner.

2) The present value of the safe 12 million Kroner is $12/1.03 = 11.7$ million Kroner.

3) With the added cash flow, the firm can now return 92 million Kroner to the creditors in the low state. The new value of debt is $(0.4(100) + 0.6(92))/1.03 = 92.4$ million Kroner. The new value of equity is $.4(52)/1.03 = 20.2$ million Kroner. The firm's value is the sum of these amounts, 112.6 million Kroner — this may also be derived as the sum of the firm value from 1) with the present value from 2).

4) The answer in 2 implies that the investment has positive net present value. However, a large share of the added cash flow will go to creditors. It is not in the interest of equity holders to invest the new 10 million Kroner if it raises the value of equity by only $.4(12)/1.03 = 4.7$ million Kroner. It is a case of under-investment due to debt overhang.

Problem 3 (Option Pricing 30%)

1) At time 0, we need to compute the probability p such that

$$20 = \frac{p30 + (1-p)\,13}{1.03},$$

solved by $p = 44.7\%$. With the corresponding method, we find at time 1 at the higher node that the probability of the up-branch in the tree is is 49.5%. Finally, at time 1 at the lower node, the probability of the up-branch is 38.5%.

2) At time 2, the values from top to bottom are dollars $(21, 0, 1, 0)$. At time 1 at the upper node, the value is

$$\frac{49.5\%\$21 + 50.5\%\$0}{1.03} = \$10.10$$

and at the lower node the value is likewise \$0.37. Exercise at time 1 would give \$9 at the upper node and \$0 at the lower node: this is worse (as expected for an American call option). Finally, at time 0 the value is $C_0 = \$4.59$ (better than exercising for \$0).

3) At time 2, the values from top to bottom are dollars $(0, 1, 0, 13)$. At time 1 at the upper node, the value of this cash flow is \$0.49 and at the lower node is \$7.76. Exercise at time 1 would give \$0 at the upper node and \$8 at the lower node. This is worse at the upper node, but better at the lower node. Thus, the option is exercised at time 1 at the lower node (if not before). At time 0, the value of not exercising is thus

$$P_0 = \frac{44.7\%\$0.49 + 55.3\%\$8}{1.03} = \$4.51.$$

This is better than exercising for \$1.

4) This present value is $\$21/\left(1.03\right)^2 = 19.79$.

5) We find $S_0 + P_0 = 24.51$ greater than $PV(K) + C_0 = 24.38$. The value of the American put option exceeds the value of its European counterpart, precisely because early exercise is optimal (at time 1 at the lower node). The put-call parity holds for European options on non-dividend paying stocks.

Problem 4 (Various Themes 25%)

1) The idea is to sketch and discuss Figure 15.2 from the textbook by Berk and DeMarzo. Under the assumptions of the Modigliani-Miller theorems, WACC is constant.

2) See chapter 6 by Ross. The existence of such a vector with $p_j > 0$ in all coordinates is equivalent to the absence of arbitrage opportunities in this market. For such a vector, we may normalize it so that its coordinates sum to 1. The equations for the assets are not changed by this. The vector can then be interpreted as a probility distribution over the m outcomes. The equations then state that each asset has zero expected gross return under this probability distribution, as if all assets are priced under risk-neutral pricing with this probability distribution.

3) See equation (30.2) in the textbook by Berk and DeMarzo and the explanation on pages 1002–3.